

Taking Math Class Notes

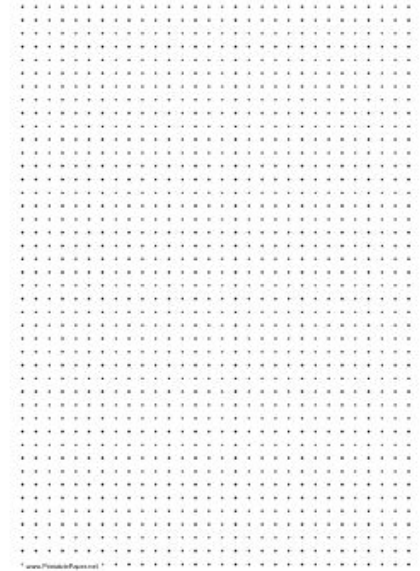
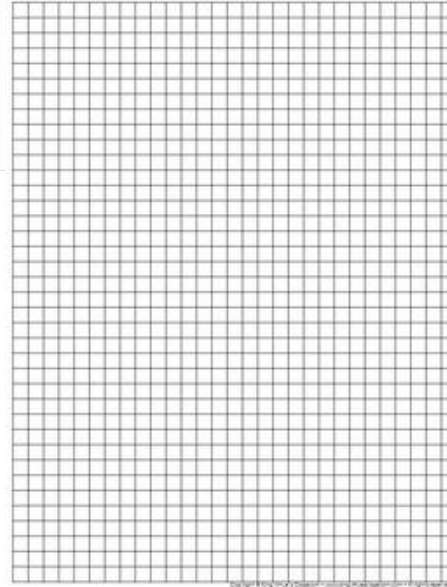
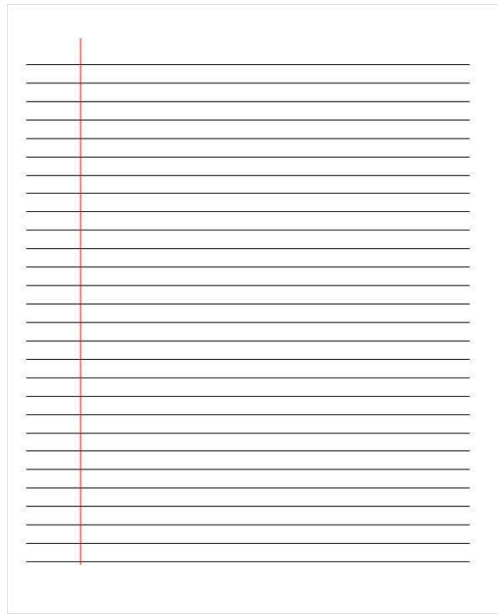
What are you currently using? How can we build on that?

Some ideas to help with organization

- Changing the type of paper you use
 - How might your formatting change if you changed paper?
 - What do you see when you look at the organization of your paper?
- Using two or more different colors as you take notes
 - Would this be useful to you?
 - Do you think you would use it? If so, in what way would you utilize it?

Paper Types

- Dotted
- Lined
- Blank
- Graphing



Try using two (or more) different color pens/pencils

- As a way to help keep your notes organized, assign a color for definitions and another for examples.

12.6 Directional Derivative
Gradient 10/29/18

Rate of change of f at (a,b) in direction \vec{u} is

$$D_{\vec{u}} f(a,b) = \lim_{h \rightarrow 0} \frac{f(a+h u_1, b+h u_2) - f(a,b)}{h}$$

Calculus a Unit vector

To compute $D_{\vec{u}} f(a,b) = \langle \frac{\partial f}{\partial x}(a,b), \frac{\partial f}{\partial y}(a,b) \rangle \cdot \langle u_1, u_2 \rangle$

Function $f(x,y) = x^2 + y^2$

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} = 2y$$

Proof: one way

$$g(s) = f(a + s u_1, b + s u_2)$$

$$g'(s)|_{s=0} = \lim_{s \rightarrow 0} \frac{g(s) - g(0)}{s - 0} = \lim_{s \rightarrow 0} \frac{f(a + s u_1, b + s u_2) - f(a,b)}{s}$$

another way

$$\frac{dg}{ds} \Big|_{s=0} = \frac{df}{ds} \Big|_{s=0} = \frac{\partial f}{\partial x} \frac{dx}{ds} + \frac{\partial f}{\partial y} \frac{dy}{ds} \Big|_{s=0} = \frac{\partial f}{\partial x}(a,b) u_1 + \frac{\partial f}{\partial y}(a,b) u_2 = \nabla f(a,b) \cdot \vec{u}$$

Rate at maximum increase: $|\nabla f(a,b)| = \text{Rate}$

Ex. What is $D_{\vec{u}} f$ at $F(1,1)$? where $\vec{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$

$$f(x,y) = x^2 + 2y^2$$

$$\nabla f(x,y) = \langle 2x, 4y \rangle \quad \nabla f(1,1) = \langle 2, 4 \rangle$$

$$D_{\vec{u}} f(1,1) = \langle 2, 4 \rangle \cdot \langle \frac{3}{5}, \frac{4}{5} \rangle = \frac{22}{5}$$

slope of the direction is $\frac{22}{5}$

Is there a direction of no change?

$$\nabla f(1,1) \cdot \vec{u} = 0$$

would be orthogonal

$$\langle 2, 4 \rangle \cdot \langle \frac{4}{\sqrt{20}}, \frac{2}{\sqrt{20}} \rangle = 0$$

In which direction \vec{u}_m is $D_{\vec{u}_m} f(1,1)$ as large as possible?

$$\langle 2, 4 \rangle \cdot \langle \frac{2}{\sqrt{20}}, \frac{4}{\sqrt{20}} \rangle = \frac{\nabla f(1,1) \cdot \nabla f(1,1)}{|\nabla f(1,1)|}$$

$$\vec{u}_m = \frac{1}{\sqrt{20}} \langle 2, 4 \rangle = \frac{\nabla f(1,1)}{|\nabla f(1,1)|}$$

$\frac{20}{\sqrt{20}} = \sqrt{20} = |\nabla f(1,1)|$ Always equal this

for all gradients $D_{\vec{u}} f(a,b) = \nabla f(a,b) \cdot \vec{u}$

The direction of steepest ascent at (a,b) is given by $\nabla f(a,b)$ and rate of change is $|\nabla f(a,b)|$

$$-\frac{\text{Decent}}{\nabla f(a,b)} \leq \frac{\text{Rate}}{\nabla f(a,b)} \leq \frac{\text{Rate}}{\nabla f(a,b)}$$

Two page/column system

- This is a way to organize the way you take notes to try to separate either examples and definitions or during and after lecture.
- Page/column one
 - This is a space to write down definitions or equations to keep it in an easy to find location.
 - Or to take all of the notes for the lecture in your organizing pattern
- Page/column two
 - This space can be used to write down all of the examples of the equations you did in class.
 - Or It can be used to rewrite your notes and develop questions and your own definitions.

Two Page System (First Page)

- Page one
 - Start by putting the date and section/chapter up at the top.
 - Use this page to write definitions, proofs, equations, and other important information
 - Personalise this page to best help you. What works best for the flow of the class and your thought process?

12.8 Critical Points 11/2/18
Max/min Values with 2 variables

If f has a local max/min at (a,b)
then $f(a,b)$ local max/min value

Local max/min call local extremum value

If $f = f(x,y)$ has a local max/min at (a,b) then
 $\nabla f(a,b) = \langle 0, 0 \rangle = \vec{0}$ ←
 (a,b) is a critical point if)
Still need to find
what critical point it is

Let $f = f(x,y)$. a point (a,b) interior to the domain of
 f is a CR. PT if

(1) $f_x(a,b) = 0$ and $f_y(a,b) = 0$
or
(2) $f_x(a,b) = \text{DNE}$ or $f_y(a,b) = \text{DNE}$

Second Derivative test
 (a,b) is a CRPT to $f = f(x,y)$

$\begin{bmatrix} f_{xx} & f_{yy} \\ f_{xy} & f_{yx} \end{bmatrix} \xrightarrow{\text{DET}} f_{xx}f_{yy} - f_{xy}f_{yx} = f_{xx}f_{yy} - (f_{xy})^2 = D(x,y)$
Same sign

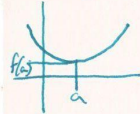
CRPT If $D(a,b) > 0 =$ local max or min
local max if $f_{xx}(a,b) < 0$ min if $f_{xx}(a,b) > 0$

Two Page System(Second Page)

- Page Two

- This page is mainly for examples, but there could be explanations as well.
- Find a way of organizing this so that you can easily find all the information.

Ex. $y = f(x)$ Conclude last possible that it is not one
 $a = \text{critical point if } f'(a) = 0$



Ex. $f(x,y) = x y(x-1)(y-4) = \begin{cases} y(y-4)(x^2-x) \\ x(x-1)(y^2-4y) \end{cases}$

$f_x = 0 \quad y(y-4)(2x-1) = 0$ 1) Start with one equation
 $f_y = 0 \quad x(x-1)(2y-4) = 0$ 2) take it case by case
3) Record Intersection

$f_x = 0 \Rightarrow y = 0 \quad y = 4 \quad x = \frac{1}{2}$
Case 1 Case 2 Case 3

Case I ($y=0$) Critical pt: $(0,0)$ and $(1,0)$
 $x(x-1)(\overset{\text{not zero}}{2(0)-4}) = 0 \Rightarrow x=0 \text{ or } x=1$

Case II ($y=4$) Critical PT: $(0,4)$ and $(1,4)$
 $x(x-1)(\overset{\text{not 0}}{2(4)-4}) = 0 \Rightarrow x=0 \text{ or } x=1$

Case III ($x=\frac{1}{2}$) CRPT: $(\frac{1}{2}, 2)$
 $\overset{\text{not 0}}{(\frac{1}{2})(\frac{1}{2}-1)(2y-4)} = 0 \Rightarrow y=2$